Modulation efficiency of double-phase hologram complex light modulation macro-pixels

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Abstract: The modulation efficiency of the double-phase hologram macro-pixel that is designed for complex modulation of light waves is defined and analyzed. The scale-down of the double-phase hologram macro-pixel associated with the construction of complex spatial light modulators is discussed.
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References and links

1. Introduction

A complex spatial light modulator (SLM) has long been sought after and would be the ideal key device for various holographic applications such as holographic microscopy, holographic sensing, and holographic three-dimensional (3D) displays [1–6]. Although the superior properties of the complex SLM are well known, at least theoretically, the complex SLM has not yet been realized in the form of one device, but tested only in the form of a bulky system.
However, recently, the development of a complex SLM has been accelerated by a strong interest in holographic applications, in particular, holographic 3D displays that are being considered as the ultimate form of 3D display [7]. Holographic 3D display could create a light field in free space that would be identical to that generated by a real 3D object, for which precise manipulation of both amplitude and phase of light wave is necessary. The informative complex number patterns displayed by an SLM that represents two-dimensional light wave modulation profile is referred to computer-generated holograms (CGHs) [8]. In the optical reconstruction of CGH contents, the use of a phase-only SLM or an amplitude-only SLM inherently produces significant optical noises such as twin noise and zeroth order non-diffracting (DC) noise. In practice, the optical noises have to be filtered out by additional optical filtering systems making the system quite bulky. In theory, though with a complex SLM, we can create realistic holographic light fields with twin, DC noises, and other encoding noises greatly reduced without additional optical filtering systems, since perfect CGH can be displayed by the original complex pattern.

The practical realization of a complex SLM is a challenging goal. Although various proposals have been reported [2–7, 9, 10], the integration of two independent control mechanisms of amplitude and phase in practical device architecture has not yet been achieved. The double-phase hologram (DPH) macro-pixel is a practical candidate for device-level complex SLM architecture. Recently, theoretical investigation on the optical imaging characteristics of DPH macro-pixels using a diffractive optic combiner was reported [9] and followed by a basic experimental study on device-level architecture [10].

The focus of this paper is the analysis of the modulation efficiency of the DPH macro-pixel and its scale-down. Modulation efficiency is a critical measure of device performance. The light wave power that does not contribute to the complex modulation in the device is lost as various optical noises. The percentage of the input light wave that is converted to the complex-modulated signal wave through the DPH macro-pixel is a fundamental problem. Also, device scaling is an important issue since the pixel pitch of the SLM determines the possible viewing angle of the holographic 3D display. Due to diffraction effect, it is inferred that there is a trade-off relationship between scale-down and modulation efficiency in the DPH structure. In this paper, the modulation efficiency of DPH macro-pixel is defined and its evaluation process is described. First, the DPH macro-pixel design with a 40$\mu$m pixel size [10] is described and its modulation efficiency is analyzed. And the 8$\mu$m down-sized macro-pixel structure is analyzed and its modulation efficiency is compared with that of the 40$\mu$m DPH structure. Both the physical issues associated with the scale-down of the DPH structure and the development direction for scaled-down complex SLMs are addressed.

2. Modulation efficiency evaluation of a DPH macro-pixel

The structure of the DPH macro-pixel are depicted in Fig. 1. The DPH macro-pixel uses a binary grating and a prism that are paired to combine the light waves from the upper and lower pixels as shown in Fig. 1(b). The working principle of the DPH macro-pixel is based on two-pixel interference. A normally incident plane wave passes through the upper and lower pixels of pitch $T_x$ with respective phase modulations, $e^{i\phi_1}$ and $e^{i\phi_2}$, and the phase-modulated light waves are refracted by the prism with a base angle $\psi$, which is designed to direct the light waves to the center of the grating plane. The two phase-modulated light waves are diffracted by a binary-phase grating that is tuned to optimally generate normally directed waves, which correspond to the positive and negative first-order diffraction waves for the upper and lower pixel, respectively. At the opposite side of the grating, the diffracted light waves propagating along the optic axis are interfered as shown in Fig. 1(b). The interferometric complex modulation can occur when the phase-modulated diffracted waves are aligned precisely. The collinear interference of the two light waves can produce a complex-modulated signal wave, $U(\phi_1, \phi_2)$, represented by the polar form,
\[ U(\phi_1, \phi_2) = 0.5 \eta \left[ \exp(j \phi_1) + \exp(j \phi_2) \right] = \eta \cos \left( \frac{\phi_1 - \phi_2}{2} \right) \exp \left[ j \left( \phi_1 + \phi_2 \right) / 2 \right], \]

where \( \eta \cos \left( \frac{\phi_1 - \phi_2}{2} \right) \) and \( \left( \phi_1 + \phi_2 \right) / 2 \) are the amplitude and phase modulation values of the complex-modulated signal wave, respectively. It can be seen that any complex modulation value can be synthesized by two independent phase variables, \( \phi_1 \) and \( \phi_2 \), if phase is modulated in a full \( 2\pi \) range.

The DPH macro-pixel is designed using the following design equations. For a given prism refraction angle \( \theta \) of a light wave, the grating period, \( \Lambda \), and the prism base angle, \( \psi \), are determined, respectively, by

\[
\Lambda = \frac{\lambda}{\sin \theta},
\]

\[
\psi = \sin^{-1} \left\{ \frac{1}{\sqrt{\frac{n_p \cos \theta}{\sin \theta}^2 + 1}} \right\},
\]

where \( \lambda \) and \( n_p \) are the wavelength of the light wave and the refractive index of the prism, respectively. The height of the prism, \( h_p \), is given by

\[
h_p = T \tan \psi.
\]

\( |\eta|^2 \) is defined to be the complex modulation efficiency, which is the power ratio of the collinearly aligned wave component represented by \( \eta \left[ \exp(j \phi_1) + \exp(j \phi_2) \right] \) to the total optical power incident to the DPH macro-pixel. Rigorous electromagnetic analysis is required to accurately evaluate the modulation efficiency. In this paper, the Fourier modal method (FMM) [11] is employed for modeling and analysis of the DPH macro-pixel and the general-purpose FMM MATLAB package included in ref [11] was used for the numerical analysis, which was performed on a laptop with Intel i7-CPU and 16GB memory.

In real situation, scattering and higher-order diffraction fields are mixed at the output plane of the grating, producing complicated field distributions. Therefore, for the evaluation of the complex modulation efficiency, we need to filter out undesired higher-order diffraction and scattering wave components to identify complex modulation component from mixed field distribution. Figure 2 concretizes the two-step extraction process of the complex-modulated signal wave component. The first step is the identification of the normally directed phase-modulated waves coming from the upper and lower phase-modulation pixels, respectively. In this step, total electric field is filtered by a simple low-pass filter in the angular spectrum domain as shown in Figs. 2(a) and 2(b). Respective filtered normal direction phase-modulated
waves are collinearly interfered in the intersection region, constructing a single complex-modulated signal wave. The overlap of two phase-modulated waves should be taken into account properly in the modulation efficiency analysis. Thus, the second step is the calculation of the intersection width, \( W_h \), as presented in Fig. 2(c). As a result, the rectangular aperture filter with width \( W_h \) is obtained. It should be noted that the complex modulation occurs only in this intersection region with width \( W_h \). In summary, the two-step filtering process is constructed as follows: the total transmitted field is low-pass-filtered in the angular spectrum domain to identify normal direction phase-modulated wave components by eliminating higher-order diffraction and scattered components and the aperture filter with width \( W_h \) is applied to extract the effective complex-modulated signal wave specified in the intersection region of the two phase-modulated waves. The modulation efficiency, \( \eta \), is measured when the maximum power flow is confined in the region. It can be also considered the percentage of the maximum power of complex-modulated signal wave to that of the input wave. The equal phase condition, \( \phi_1 = \phi_2 \), induces constructive interference of the phase-modulated waves from the upper and lower pixels and deliver maximum power flow through the complex-modulated signal wave.

3. Modulation efficiency of the 40\( \mu \)m DPH macro-pixel

The modulation efficiency of the DPH pixel with a \( T_x = 40 \mu m \) pixel pitch is analyzed using the proposed evaluation method. In the analysis, the wavelength and the refractive index of the prism are supposed to be \( \lambda = 532nm \) and \( n_{prism} = 1.47 \), respectively. The refraction angle of the light wave, \( \theta \), is set to \( \theta = 3.05^\circ \). The grating period and the prism base angle are then obtained as \( \Lambda = 10 \mu m \) and \( \psi = 6.44^\circ \) respectively from Eqs. (2a) and (2b). The height of the

![Fig. 2. Design process of the first and second filters for the modulation efficiency evaluation process.](image)

(a) Grating

Substrate

Total field distribution

Angular spectrum \( k_x \)

(b) Grating

Substrate

Total field distribution

Angular spectrum \( k_x \)

(c) Upper-pixel phase-modulated field

Lower-pixel phase-modulated field

Un-modulated field

Filtering

Angular spectrum \( k_x \)

Complex-modulated signal field

Un-modulated field

Fig. 2. Design process of the first and second filters for the modulation efficiency evaluation process. The first filter is the optical low-pass filter in the angular spectrum domain for extracting normally directed finite-size phase-modulated fields from the total field. Extraction of the normally directed phase-modulated fields from (a) the upper pixel and (b) the lower pixel. (c) The second filter is the aperture filter that constrains the spatial width of the complex-modulated wave components in the intersection area of the two filtered wave components. The modulation efficiency estimation is carried out using a Poynting vector calculation in the region specified by the second filter.
prism is set to $h_w = 4.515\mu m$. Since the modulation efficiency is proportional to the intersection region of the phase-modulated waves, the distance between the prism and the binary-phase grating should be determined to maximize the intersection region in the DPH structure. According to the condition, the distance from the prism to the binary grating is calculated by

$$h = T / (2 \cdot \tan \theta) = 373\mu m. \tag{3}$$

The filtering process is applied to this sample and followed up with field visualizations. Figures 3(a) and 3(b) present the light field distributions from the upper and lower pixels respectively in the region from the prism to the output free space that includes the binary grating with 300μm thick substrate. In Figs. 3(c) and 3(d), the angular spectrum profiles of the respective transmitted electric fields at the grating output plane are presented. As mentioned above, we can observe complicated light field distributions mixed with several higher order diffraction and internal scattering wave components in the angular spectrum domain as well as the spatial domain. In the angular spectrum profiles in Figs. 3(c) and 3(d), the highest peak is observed around the center of the transversal wavenumber axis, meaning that the normal direction phase-modulated wave is efficiently generated by the binary phase gating. In Fig. 3(e), the transversal amplitude profiles of the phase-modulated waves from the upper and lower pixels are indicated with red and green solid lines. The intersection area of the two waves is approximately measured with width of $W_h = 58.4\mu m$ and the rectangular aperture with the same width is taken as the second filter.

In Figs. 4(a) and 4(b), the total field distributions under the equal-phase condition, $\phi_1 = \phi_2$, and the filtered complex-modulated signal wave are presented, respectively. In the first step of the filtering process, the angular spectrum of the total field specified within the narrow-band around $k = 0$ is exclusively selected. The filtered angular spectrum profiles are indicated by a red-line curve in Fig. 4(c). The transversal amplitude envelope of the normally directed field profile that is reconstructed from the filtered angular spectrum is plotted in Fig. 4(e).
Fig. 4. Identification of the complex-modulated signal wave using the two step filtering process: (a) total electric field distribution, (b) filtered complex-modulated signal wave, (c) the first filtering process to eliminate higher-order diffraction components, and (d) the second filtering process to identify the complex-modulated signal wave in the intersection region of width $W_h$. 

4(d) with the aperture filter of width $W_h$. This two-step filtering enables the identification of the complex-modulated signal wave as shown in Fig. 4(b). The finite-size beam propagating along the normal direction is the wave component that we can completely control in the manner of complex modulation with independent variables, $\phi_1$ and $\phi_2$. The amplitude, phase, and optical power of this signal wave can be calculated numerically.
Fig. 5. Complex modulations and corresponding light field distributions in the 40μm DPH macro-pixel for (a) $U(\phi_1 = 350^\circ, \phi_2 = 170^\circ) = 0$, (b) $U(\phi_1 = 170^\circ, \phi_2 = 240^\circ) = 0.67 + j0.02$, and (c) $U(\phi_1 = 20^\circ, \phi_2 = 20^\circ) = -0.82 + j0.04$. The filtered complex-modulated signal field distributions are presented together with the corresponding total field distributions for all cases.

The status of the complex modulation of the light wave can be represented in a polar coordinate diagram. The filtered complex-modulated signal field distributions for three different complex modulations, $U(\phi_1, \phi_2)$: (i) $U(350^\circ, 170^\circ) = 0$, (ii) $U(170^\circ, 240^\circ) = 0.67 + j0.02$, and (iii) $U(20^\circ, 20^\circ) = -0.82 + j0.04$ and their polar coordinate diagrams are presented in Figs. 5(a)-5(c), respectively, with their corresponding total field distributions. The amplitude and phase of the complex-modulated signal wave are measured directly from the wave pattern. It is noteworthy in Fig. 5(a) that almost-complete dark amplitude modulation is achievable in the DPH pixel architecture. In the case of this zero-amplitude modulation, most of optical power is distributed to higher order diffraction wave components. Since zero-amplitude modulation is a type of destructive resonance, the back reflection of light waves and higher-order diffraction waves become intensified. The intensified higher-order diffraction waves that produce optical noises are thought to degrade image quality in a perspective of display applications. To resolve this noise reduction problem, a strategy for absorbing the undesired spatial optical noises should also be devised in the device architecture. When observers' positions are restricted in the far-field region, the observers can see only the complex-modulated signal wave components without contamination by higher-order diffraction noise components.

The full dynamic range of the complex modulation that can be achieved by the 40μm pixel structure is analyzed by changing the phase modulation values, $\phi_1$ and $\phi_2$ in the full range from 0 to $2\pi$. The phase and amplitude modulations are plotted in Figs. 6(a) and 6(b), respectively, in terms of $x = (\phi_1 - \phi_2)/2$ and $y = (\phi_1 + \phi_2)/2$. The amplitude and phase modulations measured numerically from the light field distribution accord well with those
given in Eq. (1), \( \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \) and \( \arg\left(\exp\left[j\left(\phi_1 + \phi_2\right)/2\right]\right) \), respectively, where \( \arg(\alpha) \) means the phase of a complex number \( \alpha \). The full dynamic range of complex modulation achieved in this analysis is represented in the polar coordinate diagram of Fig. 6(c). It confirms that the DPH pixel covers the full complex modulation range. As seen in Fig. 6(a), the zero-amplitude and maximum amplitude modulations are obtained with the conditions, \( |\phi_1 - \phi_2| = \pi \) and \( |\phi - \phi_2| = 0 \), respectively.

![Fig. 6. Modulation dynamic range of the DPH macro-pixel: (a) amplitude, (b) phase, and (c) its polar coordinate diagram representation.](image)

The modulation efficiency of the DPH macro-pixel is calculated by using the Poynting vector of the signal light wave under the equal phase condition. Here, we compare the modulation efficiencies of the DPH macro-pixels with the optimal distance \( h_1 = 373\mu m \) (see Eq. (3)) and a smaller distance \( h_1 = 270\mu m \), for which a reduction in modulation efficiency is expected. The modulation efficiencies for the distances, \( h_1 = 373\mu m \) and \( h_1 = 270\mu m \) are listed in Table 1. The ratios of the signal power to the transmitted power for \( h_1 = 373\mu m \) and \( h_1 = 270\mu m \) are analyzed to 70.30% and 55.91%, respectively, which can be interpreted as signal to noise ratio. The loss is mainly the result of a reduction in the intersection area reduction of the two normal-direction light waves. Interestingly, the difference in the modulation efficiency, 54.02% for \( h_1 = 373\mu m \) and 50.58% for \( h_1 = 270\mu m \) is not very large because the reflection efficiency for \( h_1 = 270\mu m \) is quite smaller than that for \( h_1 = 373\mu m \). It appears that the DPH structure configures an effective optical resonator having an internal light-circulating loop. The reflection and transmission efficiencies are dependent on \( h_1 \). In the design, we need to add an anti-reflection coating to reduce the surface-reflectance of the structure and to prevent the resonance effect so as to enhance the modulation efficiency. Conclusively, we have obtained a modulation efficiency of about 54% for the 40\( \mu m \) pixel architecture.

### Table 1. Quantitative Modulation Efficiency Evaluation of the 40\( \mu m \) DPH Macro-Pixel.

<table>
<thead>
<tr>
<th>( h_1 )</th>
<th>( W_0 )</th>
<th>Reflected wave</th>
<th>Transmitted wave</th>
<th>Complex-modulated signal wave</th>
<th>Percentage to the input power</th>
<th>Percentage to the input power</th>
<th>Percentage to the transmitted power</th>
<th>Percentage to the input power</th>
</tr>
</thead>
<tbody>
<tr>
<td>373( \mu m )</td>
<td>58.4( \mu m )</td>
<td>23.16%</td>
<td>76.84%</td>
<td>54.02%</td>
<td>70.30%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>270( \mu m )</td>
<td>32( \mu m )</td>
<td>9.53%</td>
<td>90.47%</td>
<td>50.58%</td>
<td>55.91%</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### 4. Scale-down to an 8\( \mu m \) DPH macro-pixel

The scale-down of the DPH macro-pixel is essential for its practical use in complex SLMs. Eventually, the pixel pitch should be reduced to the wavelength scale in order to achieve high quality holographic displays. Intensive study on the scale-down is necessary.

Here, through a comparison of the architectures with an 8\( \mu m \) pixel pitch and a 40\( \mu m \) pixel pitch, the issues related to the scale-down will be discussed. In Fig. 7(a), the 8\( \mu m \) -DPH...
architecture with design parameters denoted is presented. The prism refraction angle is set to a relatively large angle of 30°, with which higher-order diffractions can be damped into the evanescent region, and the prism base angle is set to $\psi = 31.62^\circ$. The binary-phase grating period and its diffraction efficiency are $\Lambda = 1.064 \mu m$ and 51.45%, respectively, as shown in Fig. 7(b).

The field distributions for the cases of turning on only the upper pixel, only the lower pixel, both the upper and lower pixels, and the complex-modulated signal wave are presented in Figs. 7(c)-7(f), respectively. In Fig. 8, the complex modulation is represented in the polar coordinate diagram. It is noted that the $8 \mu m$ DPH architecture succeeds in representing zero-amplitude modulation. For a complex modulation, the total light field and filtered complex-modulated signal field are presented together for clear visualization of device operation. In Table 2, a few data related to the modulation efficiency of the $8 \mu m$ DPH structure are listed. Modulation efficiency to the input power is analyzed to 37.05% for the case of the binary-phase grating, and the percentage to the transmitted wave is 63.92%. From Tables 1 and 2, the total transmission powers of the 40 $\mu m$ and $8 \mu m$ macro-pixels are measured to 76.84% and 57.96%, respectively. Those for complex modulation are 54.02% and 37.05%, and the percentages to the transmitted power powers are 70.30% and 63.92%, respectively.

The scale-down leads to a considerable decrease in the modulation efficiency, but we can see the possibility of pushing the DPH to even smaller scale from this analysis. The magnitude of reduction magnitude in the percentage to the transmitted power is relatively small compared with that in the total transmission efficiency. Therefore, if we can enhance the total transmission efficiency, a practical scaled-down DPH structure can be produced. Anti-reflection coating and structural optimization seems to be crucial to achieve the scaled-down DPH with high modulation efficiency. Besides reduction in modulation efficiency, the fundamental limitation of the scale-down is that the grating period cannot be scaled down for a fixed refraction angle. Thus, the total dimension of the DPH device cannot be smaller than the grating period. The pixel pitch that covers two or more periods of the diffraction grating ($= 2 \sim 3 \mu m$ for wavelength 532nm) would be a necessary condition for proper operation of a complex modulation macro-pixel. To determine the lower bound of the DPH scale that breaks down the complex modulation function, further study is required.
Fig. 8. Complex modulation and corresponding light field distributions in the 8μm DPH macro-pixel for (a) \( U(\phi_1 = 350^\circ, \phi_2 = 170^\circ) = 0 \), (b) \( U(\phi_1 = 170^\circ, \phi_2 = 240^\circ) = 0.57 + j0.47 \), and (c) \( U(\phi_1 = 130^\circ, \phi_2 = 130^\circ) = 0.73 - j0.53 \).

Table 2. Quantitative modulation efficiency evaluation of the 8μm DPH macro-pixel

<table>
<thead>
<tr>
<th>Reflected wave</th>
<th>Transmitted wave</th>
<th>Complex-modulated signal wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage to the input power</td>
<td>Percentage to the input power</td>
<td>Percentage to the input power</td>
</tr>
<tr>
<td>42.04%</td>
<td>57.96%</td>
<td>37.05%</td>
</tr>
</tbody>
</table>

With the scaling-down issue, the fill-factor limitation due to the finite intersection width of the DPH macro-pixel requires an elaborate study. In theory, the SLM fill-factor smaller than 100% of the SLM causes the generation of non-diffracting component, i.e. DC noise in the diffraction field or holographic reconstruction field. In practice, since the SLM architecture with 100% fill-factor is difficult to fabricate, we should always assume a finite fill-factor. However, if the footprint of the SLM becomes on a scale comparable to wavelength, the DC noise can be avoided by the well-known off-axis hologram configuration, where the DC is separated from the signal field. If the DPH SLM is filtered by a simple Fourier filter, for example, a 4-f system with a DC rejection filter, the effective SLM obtained after the filtering process can become one with near 100% fill-factor [12]. The dead-area of the DPH pixel is blurred to become a homogenized pixel. But the design of thin compact integrated filtering structure is a challenging issue. The scaling-down and fill-factor issues are tightly associated and have to be dealt with on the same framework together.

5. Conclusion

In conclusion, we have analyzed the complex-modulation efficiency of the DPH macro-pixel structure and discussed the scale-down problem of the DPH macro-pixels. The modulation efficiencies of the 40μm and 8μm-macro-pixels were evaluated to be 54.02% and 37.05%, respectively. It is expected that the DPH structure for an operating wavelength of 532nm can be achieved on a scale of 2~3μm with optimal antireflective coating and structural optimization. This device structure could be a fundamental element for thin complex SLM optimized for holographic 3D display. In a broad view, the development of complex SLM might be the most impact technology in the field of wave optic engineering.
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